HIGH-RESOLUTION FREQUENCY-WAVENUMBER SPECTRA

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HIGH-RESOLUTION FREQUENCY-WAVENUMBER SPECTRA

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ABSTRACT

The method of high-resolution spectral analysis has been applied to one- and two-dimensional wavenumber spectra. In this method a prediction operator is used to extend the length of the spatial correlation functions in order to obtain higher resolution of the resulting spectra in wavenumber space. It is most useful in the case of small arrays, where the spatial correlation functions are too short to give adequate resolution for the ordinary frequency-wavenumber spectra. A theoretical description of the method, its procedures, and its relation to ordinary wavenumber spectra is given. In particular, an analytical expression is derived for the resolution and the response of the process in the case of an input plane wave, where the high-resolution method is averaged over all reference sensors.

Disadvantages of the high-resolution method are discussed, in particular an unrecoverable distortion of the true amplitude spectrum. Finally, examples are given for data recorded at the Wichita Mountains Seismic Observatory surface array and the Apache, Oklahoma, vertical array.

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INTRODUCTION

Burg (1967) has shown that high-resolution power spectra can be computed from relatively short correlation functions by first least-squares predicting larger correlation lags from those already known. The resulting high-resolution power spectrum is inversely proportional to the power spectrum of the corresponding prediction error operator. However, with the advent of the Cooley-Tukey fast Fourier transform algorithm and related methods for estimating power spectra, the high-resolution technique for ordinary power spectra is no longer economical. It is now feasible to compute power spectra as if they were Fourier transforms of the 100%-lag correlation functions; various kinds of smoothing can then be applied to reduce the spectra to the desired length and stability.

We feel that the most promising application of the high-resolution procedure lies in estimating frequency-wavenumber spectra. Here the length of the spatial correlation functions, which ultimately determines the resolution, is limited by the size of the array. The high resolution method is therefore the appropriate way to obtain frequency-wavenumber spectra which are sharp in wavenumber space from data recorded over small arrays.

Ordinary frequency-wavenumber spectra (Spieker et al., 1961) are computed directly from the spectral matrix elements by the expression:

$$P(\omega,\underset{\sim}{k}) = \sum_{i=1}^{N} \sum_{j=1}^{N} S_{ij}(\omega) \exp\left[-i\underset{\sim}{k}\cdot(\underset{\sim}{x}_i - \underset{\sim}{x}_j)\right] \tag{1}$$

In the case of two-dimensional arrays (for example, LASA) P is a function of three variables: ω, $\underset{\sim}{k}_x$, and $\underset{\sim}{k}_y$. The standard way of displaying this function is to produce a succession of contour plots, each at a particular frequency, with P contoured against $\underset{\sim}{k}_x$ and $\underset{\sim}{k}_y$. On the other hand, for one-dimensional

arrays (for example, arrays of instruments in deep wells) only the vertical wavenumber is involved and P can be contoured against ω and k_z in one plot. Loci of constant velocity

$$\underset{\sim}{v} = \frac{\omega \underset{\sim}{k}}{\underset{\sim}{k} \cdot \underset{\sim}{k}} \tag{2}$$

are circles about the $\underset{\sim}{k}_x$, $\underset{\sim}{k}_y$ origin as shown in Figure 1a or lines of constant slope as shown in Figure 1b.

A measure of the resolution or capability of the array is its response to an infinite-velocity plane wave. If the instrument frequency response is neglected, the spectral matrix of the wave is constant. Therefore this function, usually called the array response function (Spieker et al., 1961), is given by:

$$R(\underset{\sim}{k}) = \sum_{i=1}^{N} \sum_{j=1}^{N} \exp\left[-i\underset{\sim}{k}\cdot(\underset{\sim}{x}_i - \underset{\sim}{x}_j)\right] \tag{3}$$

Any single finite-velocity plane wave of the form

$$\exp\left[i(\underset{\sim}{k}_o \cdot \underset{\sim}{x} - \omega t)\right] \tag{4}$$

will produce a frequency-wavenumber spectrum consisting of the array response function centered on the value $\underset{\sim}{k}_o$ modulated in frequency by the corresponding frequency power spectrum.

The high-resolution technique consists of using the spectral matrix of the data to design a multichannel prediction error operator. High-resolution frequency-wavenumber spectra are then computed from the spectral matrix of this multichannel filter (Haney, 1967). The appropriate matrix equation is:

$$\left[S(\omega) + cI\right] \underset{\sim}{f}^m(\omega) = \underset{\sim}{g}^m \tag{5}$$

Here $S(\omega)$ is the spectral matrix of the data, I is an N-by-N identity matrix, $\underset{\sim}{f}^m(\omega)$ is the filter vector, and $\underset{\sim}{g}^m$ is the righthand side vector consisting of N-1 zeros and a one in the $m^{\underline{th}}$ row. This equation defines the filter which is optimum, in the least-squares sense, for "whitening" the $m^{\underline{th}}$

channel. Another way of interpreting equation (5) is to recognize it as a multichannel Wiener-Hopf equation with $S(\omega)$ the noise spectral matrix, cI the signal model, and $\underset{\sim}{g}^m$ proportional to the cross spectra between the desired output, white noise, and the input. The constant c is a convenient means of adjusting the signal-to-noise ratio of the input.

The spectral matrix of the filter is then

$$H^m(\omega) = E \left[\underset{\sim}{f}^m(\omega) \underset{\sim}{f}^m(\omega)^+ \right] \tag{6}$$

where the dagger indicates Hermitian conjugation and the brackets specify that expected values are to be taken. High-resolution frequency-wavenumber spectra of the original data channels are inversely proportional to the ordinary frequency-wavenumber spectra of this spectral matrix:

$$P'_m(\omega,k) = \left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} H^m_{ij}(\omega) \exp \left[-i\underset{\sim}{k}\cdot(\underset{\sim}{x}_i - \underset{\sim}{x}_j) \right] \right\}^{-1} \tag{7}$$

In practice, we have found that high-resolution frequency-wavenumber spectra computed this way (from single-output "whitening" filters) sometimes suffer an angular distortion (Lintz, 1968). In other words, the resolution in wavenumber space is a function of $\underset{\sim}{k}-\underset{\sim}{k}_o$. Furthermore, it is undesirable to have the technique depend on the choice of the reference data channel. This difficulty can be circumvented by using the average spectral matrix of the filters, H, computed by averaging over all reference data channels. This procedure is also a convenient means for taking the expected value.
Thus:

$$H(\omega) = \sum_{m=1}^{N} H^m(\omega) = \sum_{m=1}^{N} \underset{\sim}{f}^m(\omega) \underset{\sim}{f}^m(\omega)^+ \tag{8}$$

Since each filter vector $\underset{\sim}{f}^m(\omega)$ is merely the $m^{\underline{th}}$ column of the

inverse matrix:

$$\left[S(\omega) + cI\right]^{-1} \tag{9}$$

the required averaged spectral matrix $H(\omega)$ is easily shown to be:

$$H(\omega) = \left[S(\omega) + cI\right]^{-2} \tag{10}$$

This modification of the original procedure is used for the examples given below. It should be noted that computing high-resolution frequency-wavenumber spectra this way is only negligibly more expensive than computing ordinary frequency-wavenumber spectra. Both techniques require computing the spectral matrix. The only other computations required by the high-resolution technique are the addition of some white noise, one Hermitian matrix inversion, and one Hermitian matrix multiplication.

The actual improvement in resolution gained by using the averaged high-resolution technique can be demonstrated by applying it to a plane wave. Neglecting the frequency part, the spectral matrix of this wave is:

$$S_{ij}(\omega) = \exp\left[i\underset{\sim}{k}_o\cdot(\underset{\sim}{x}_i-\underset{\sim}{x}_j)\right] \tag{11}$$

which can be factored into the outer product of two vectors:

$$S(\omega) = \underset{\sim}{q}\,\underset{\sim}{q}^+ \tag{12}$$

where the vector $\underset{\sim}{q}$ is given by:

$$\left[\underset{\sim}{q}\right]_i = \exp\,(i\underset{\sim}{k}_o\cdot\underset{\sim}{x}_i) \tag{13}$$

The matrix to be inverted is:

$$(\underset{\sim}{q}\,\underset{\sim}{q}^+ + cI) \tag{14}$$

which is equal to:

$$\frac{1}{c}\left(I - \frac{\underset{\sim}{q}\,\underset{\sim}{q}^+}{c+N}\right) \tag{15}$$

Squaring this matrix gives, to terms of order c^3,

$$\frac{1}{c^2}\left[I - \frac{2S}{c+N} + \frac{NS}{(c+N)^2}\right] = \frac{1}{c^2}\left[I-S\left(\frac{1}{N} - \frac{c^2}{N^3}\right)+ 0(c^3)\right] \tag{16}$$

Substituting equation (16) into equation (7) gives the result:

$$P'(\underset{\sim}{k}) = \frac{c^2}{N - P(\underset{\sim}{k})\left(\frac{1}{N} - \frac{c^2}{N^3}\right)} \qquad (17)$$

Now when $P(\underset{\sim}{k})$ reaches a maximum value of N^2, $P'(\underset{\sim}{k})$ has the value N.

However when $P(\underset{\sim}{k})$ has fallen off to N^2/a, $P'(\underset{\sim}{k})$ is:

$$P'(\underset{\sim}{k}) = \frac{c^2}{N - \frac{N}{a} + \frac{c^2}{Na}} \qquad (18)$$

which, by choosing c to be much less than N;

$$c << N \qquad (19)$$

makes $P'(\underset{\sim}{k})$ very small indeed. The resolution of this technique therefore is dependent upon both N, the number of data channels, and the constant c.

PROCEDURES

All of the spectral matrix elements computed for the examples given below were done so by the Cooley-Tukey method (McCowan,1967). The data was Fourier transformed together with a sequence of zeros equal to its entire length and then cross-multiplied to produce the raw auto- and cross-spectra. These raw spectra were then subjected to a number of smoothing operations. Each operation consisted of convolving the spectra with the three-point operator (1/4, 1/2, 1/4) and taking every other point of the result. The effects of this window are described elsewhere (McCowan, 1967). Typically the original data channels were of the order of 4096 digital points for noise samples and 256 digital points for signals, both sampled at twenty points per second. This gave 4097-point and 257-point raw spectra, respectively, which were then smoothed and reduced to 65 and 33 actual frequency values respectively. Thus the number of degrees of freedom included in each spectral estimate was 64 in the case of noise samples and 8 in the case of signals.

Both to alleviate the problem caused by instrument gain anomalies (Baldwin, 1964) and the question of physical units in choosing the constant c in equation (5), we have used normalized spectral matrices (coherence matrices) throughout. Our spectral matrix elements are thus:

$$S_{ij}(\omega) = \frac{E\{F_i(\omega)F_j(\omega)^*\}}{E\{|F_i(\omega)|^2\}\ E\{|F_j(\omega)|^2\}} \tag{20}$$

Otherwise, choosing the constant c, and hence determining the resolution of the technique, would depend on the actual magnitude of the auto-spectra.

In the examples below we have chosen frequency in cps instead of angular frequency as an independent variable. Consequently, we have re-defined the wavenumber $\underset{\sim}{k}$ as:

$$|\underset{\sim}{k}| = \frac{1}{\lambda} \tag{21}$$

so that equation (2) now reads:

$$\underset{\sim}{v} = \frac{f\underset{\sim}{k}}{\underset{\sim}{k} \cdot \underset{\sim}{k}} \tag{22}$$

This was done purely as a matter of convenience in reading the countour plots.

Finally, the contour plots displayed below were computed on a grid of 21-by-21 $\underset{\sim}{k}_x$ and $\underset{\sim}{k}_y$ values for the surface array and 21-by-33 $\underset{\sim}{k}_z$ and frequency values for the vertical array. Both were then linear-interpolated in each direction, to a 100-by-60 grid to be printed by the computer printer. These pages were then traced to produce the figures. Any details smaller than 1/20 of the whole figure are therefore strongly colored by the linear interpolation.

EXAMPLES

Two events (P wave signals summarized in Table 1) and samples of noise immediately preceding them were selected as examples of high-resolution frequency-wavenumber data processing. The first event and its corresponding noise sample were used to illustrate the application of the technique to a two-dimensional surface array, in this case the WMSO array, while the remaining data were used to illustrate results from a one-dimensional vertical array, the APOK deep well. Both arrays are descibed by the coordinates of their elements, which are listed in Table 2, and their array response functions, which are shown as contour-plots in Figures 3 and 8 respectively. A map of the WMSO array is included as Figure 2.

Ordinary frequency-wavenumber spectra of both pairs of noise samples and signals are shown in Figures 4, 6, 9a, and 10a. Figures 4 and 9a are the noise samples and Figures 6 and 10a are the signals. The WMSO noise seems to be coming from the northeast with a velocity ranging between 3 and 6 km/sec. This characteristic behavior has been noted by other investigators (Rizvi et al., 1967). On the other hand, the WMSO signal appears to be coming from the south or southeast with a velocity somewhere between 16 and 20 km/sec. The APOK noise sample displays a tilted pattern also characteristic of the site. One explanation for this phenomenon is suggested by the geological structure in the vicinty of the well (Geotechnical Corp., 1964), which consists of Paleozoic intrusives and sediments dipping 30-40° towards the northeast. The APOK signal is very strange indeed, composed of a wave travelling down the well with an apparent vertical velocity of approximately 3.5 km/sec and a horizontally travelling wave with infinite vertical velocity. One possible explanation of this result is again suggested by the dip of the structure (Sax, 1967). The incoming P wave might be refracted by the tilted interface so that the wave front would be a leaking suface wave when it reached the well.

High-resolution frequency-wavenumber spectra are shown for all these data samples in Figures 5, 7, 9b, and 10b. These were computed with a signal-to-noise ratio of 2, which seemed to be a good balance between high resolution and excessive distortion. For the WMSO examples, the technique decreased the width of the -3db contour by more than a factor of two for both the cases of noise and signal. The results are more difficult to assess for the vertical array data because of the "whitening" in frequency done by the filters. However, for the noise sample at 0.25 cps, a decrease by a factor of two in the width of the -3 db contour can again be seen.

A marked disadvantage and a difficulty still to be solved in the application of this technique is illustrated in Figure 10b. The high-resolution technique, when applied to short samples of data, can produce spurious peaks in the frequency-wavenumber spectra. These are due to the limited smoothing done when computing spectra from short samples of data. Figure 10b shows several of these peaks. This is regarded as an unrecoverable distortion and in a certain sense, an inevitable result of using a high-gain procedure.

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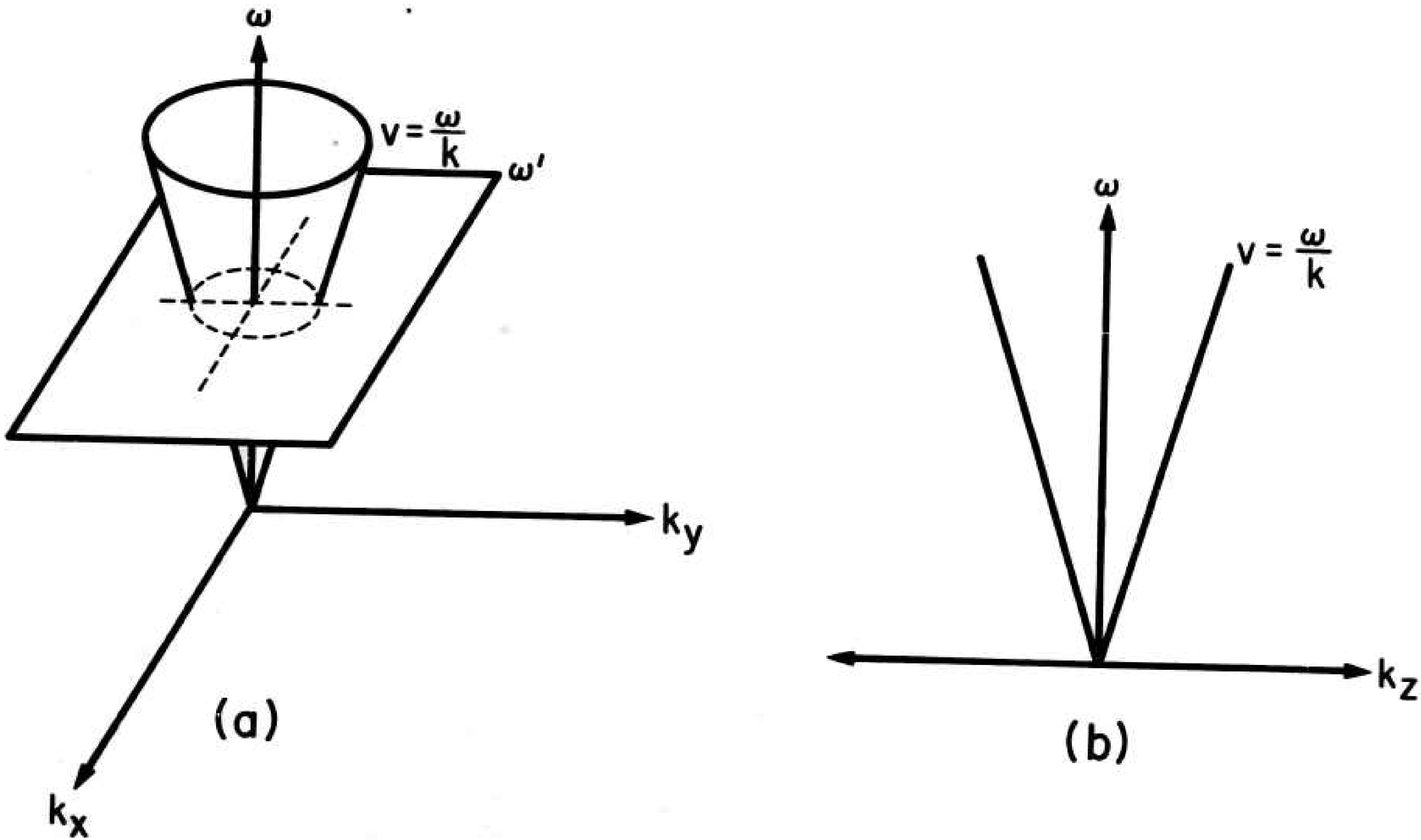

Figure 1. Constant velocity loci in frequency-wavenumber space

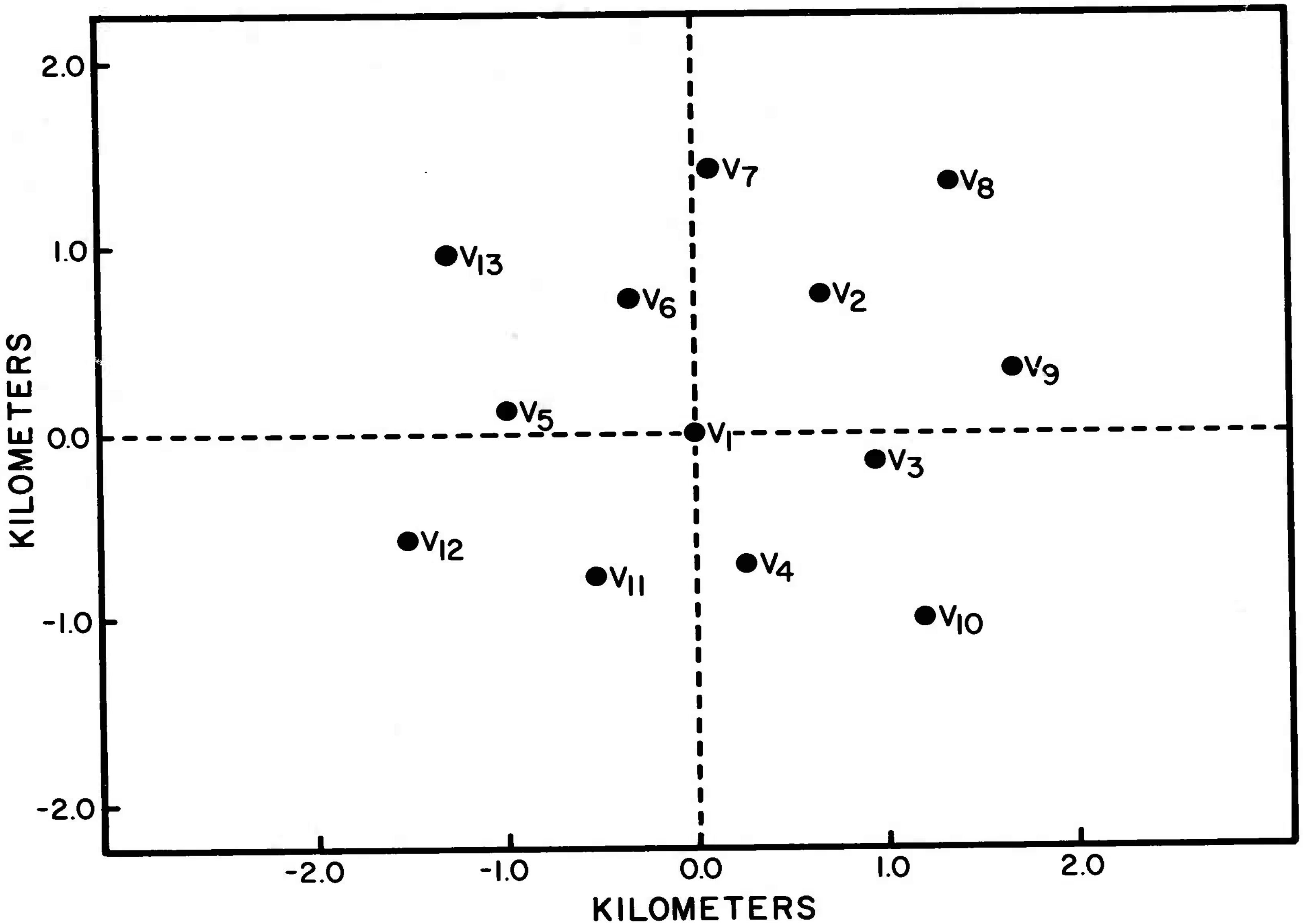

Figure 2. WMSO surface array

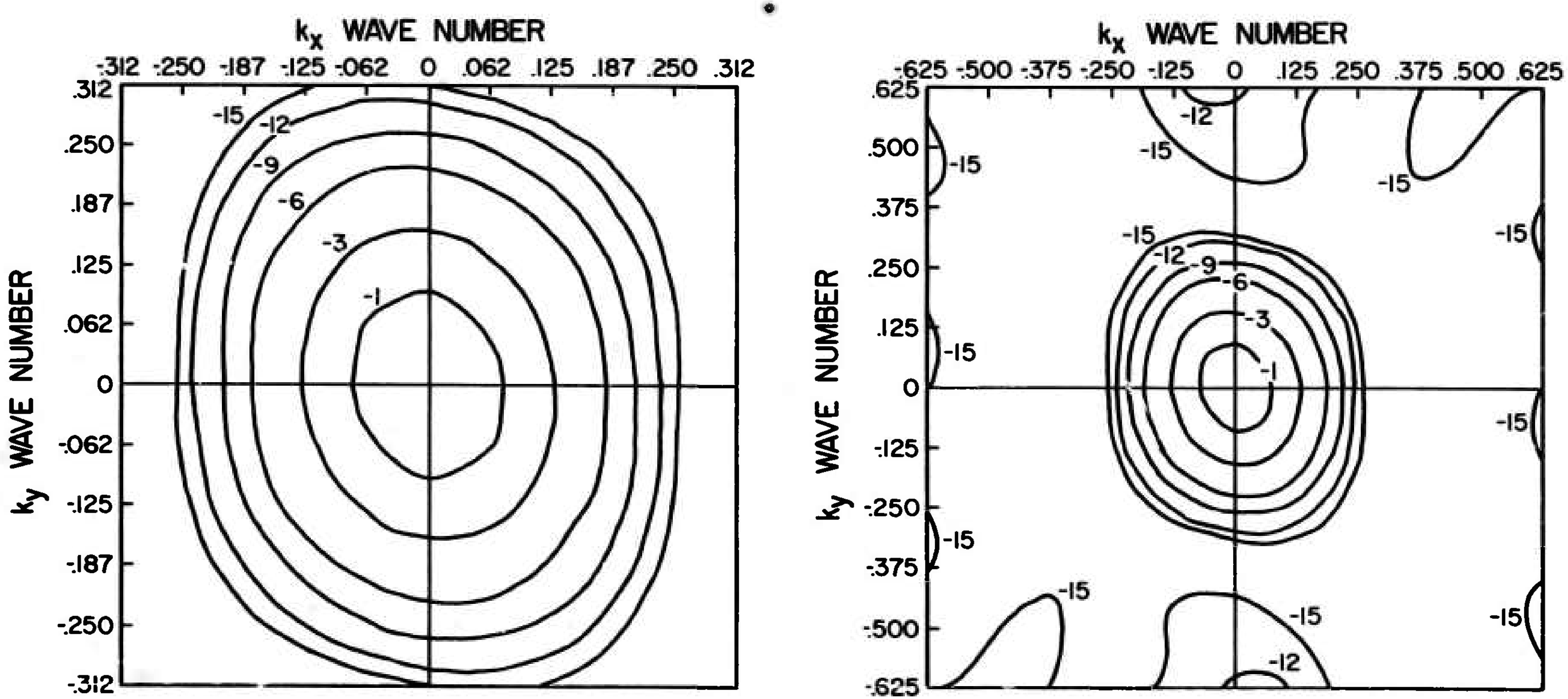

Figure 3. WMSO array response

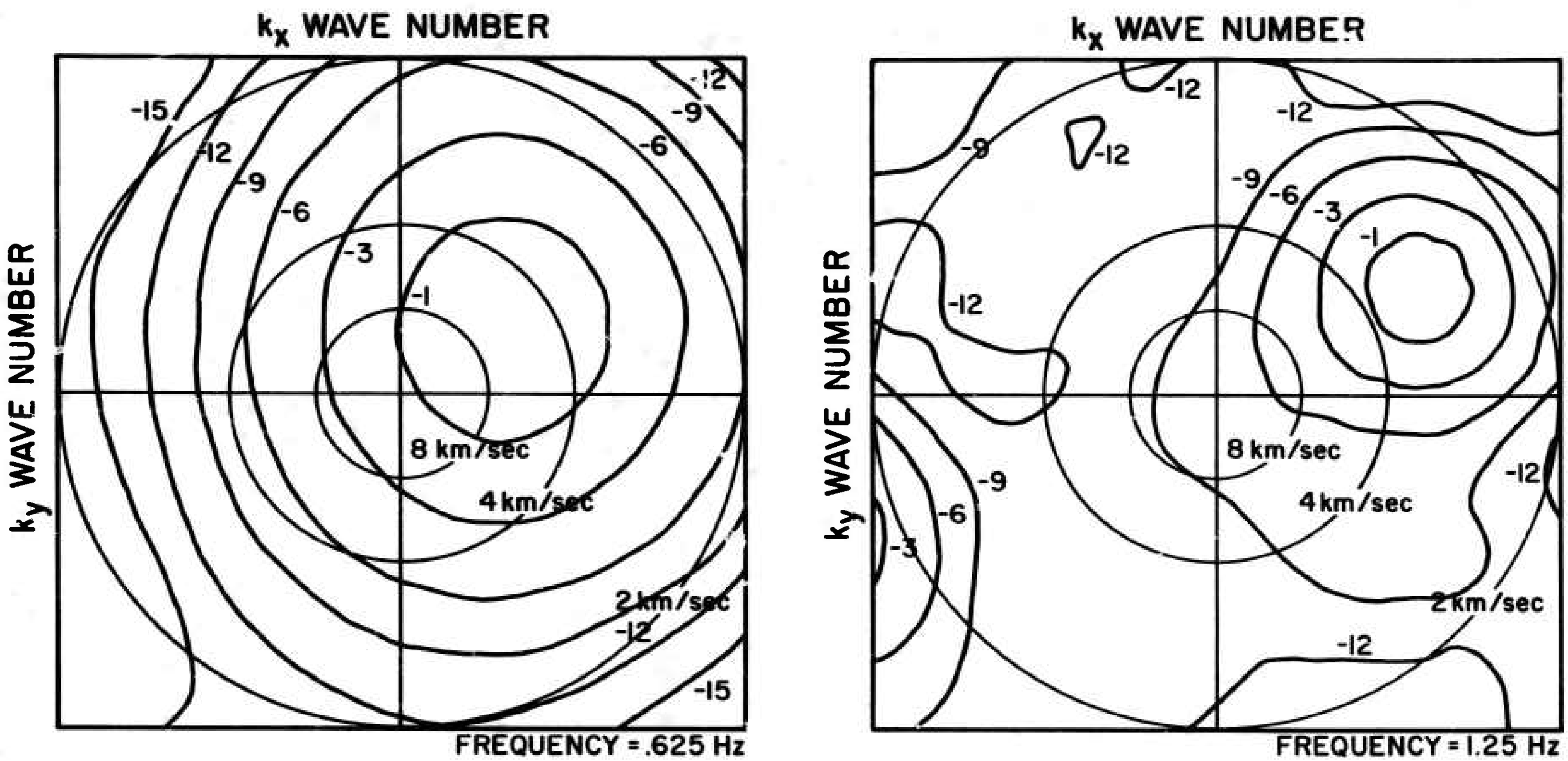

Figure 4. Ordinary frequency-wavenumber spectra of WMSO noise

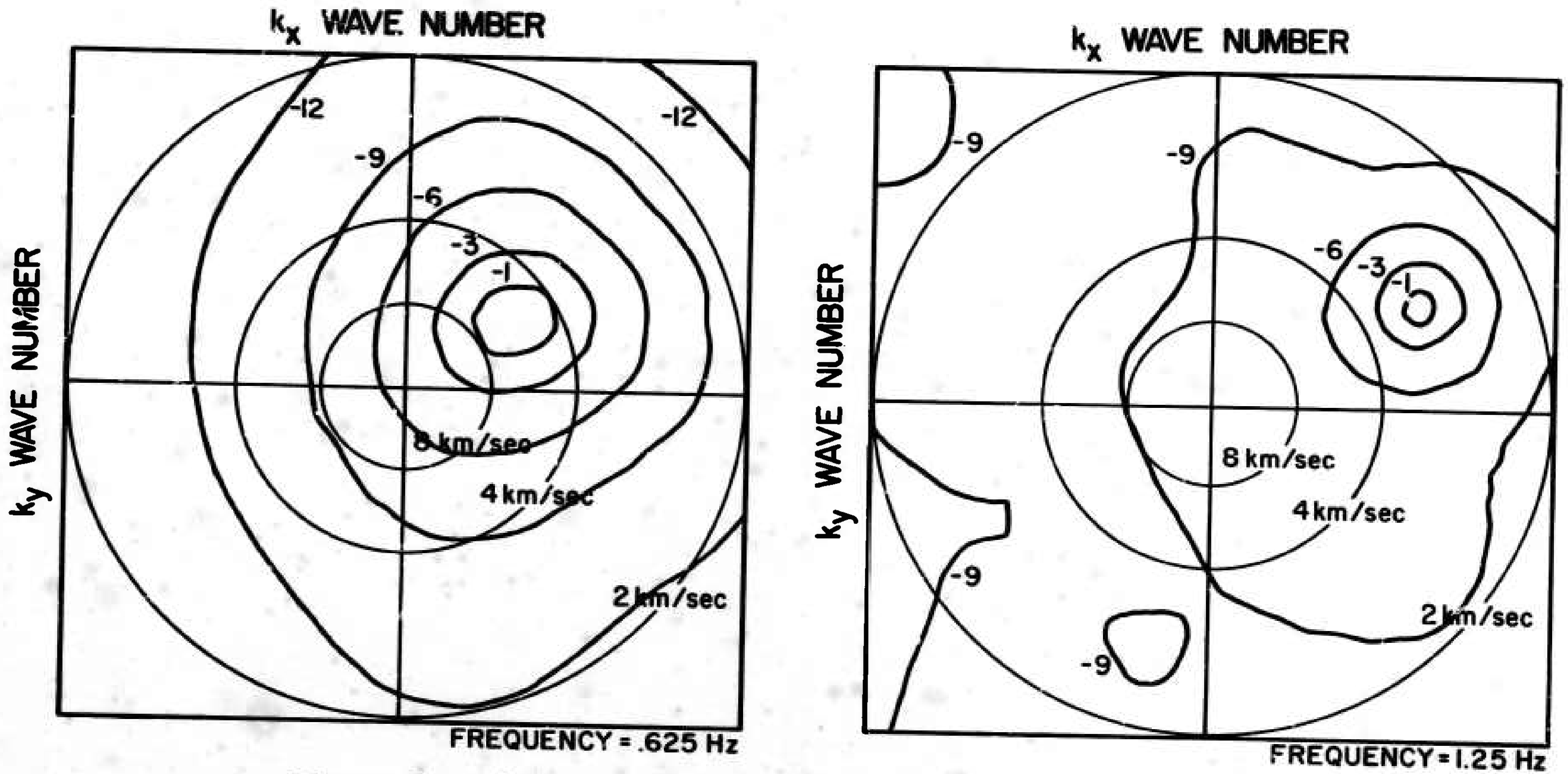

Figure 5. High-resolution frequency-wavenumber spectra of WMSO noise

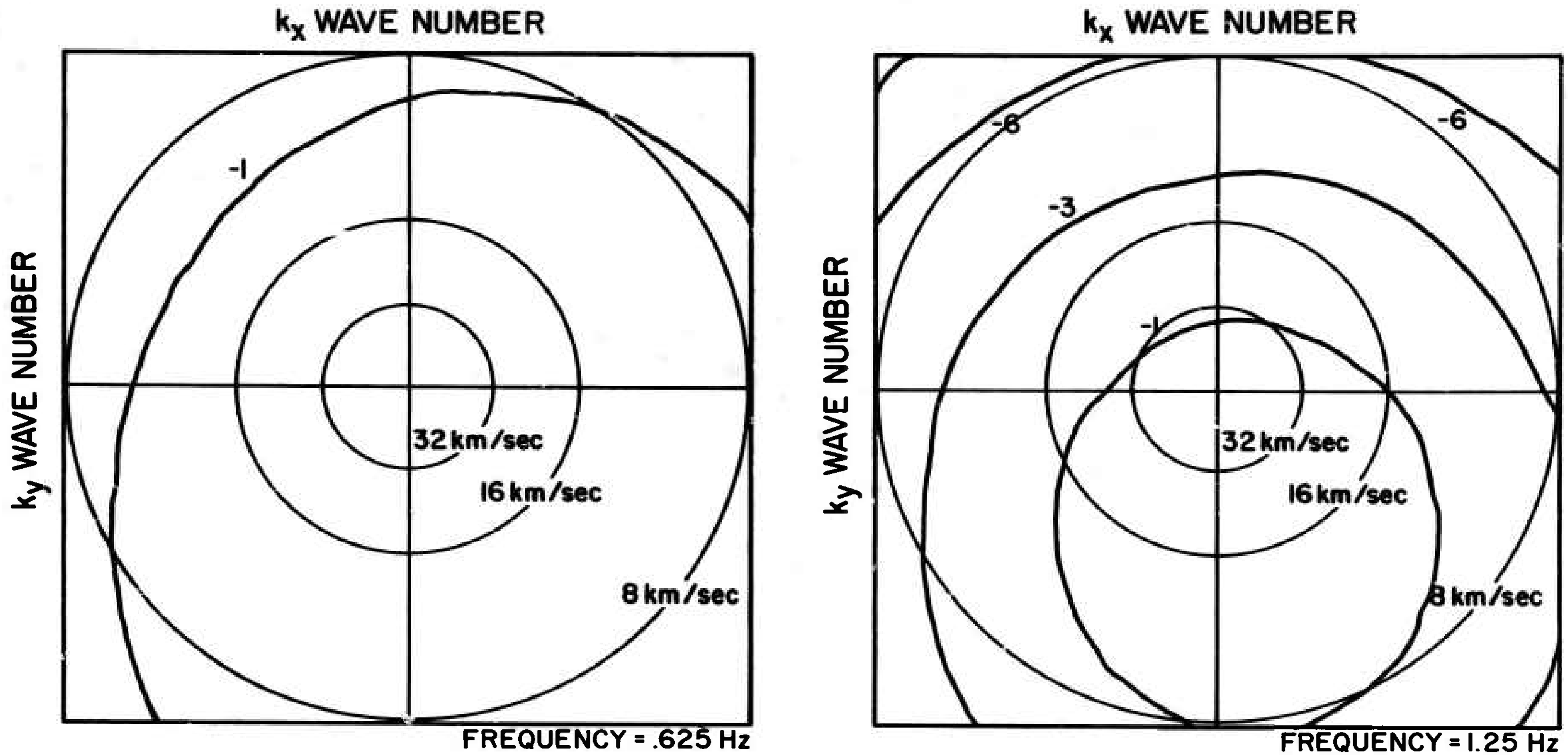

Figure 6. Ordinary frequency-wavenumber spectra of a WMSO signal

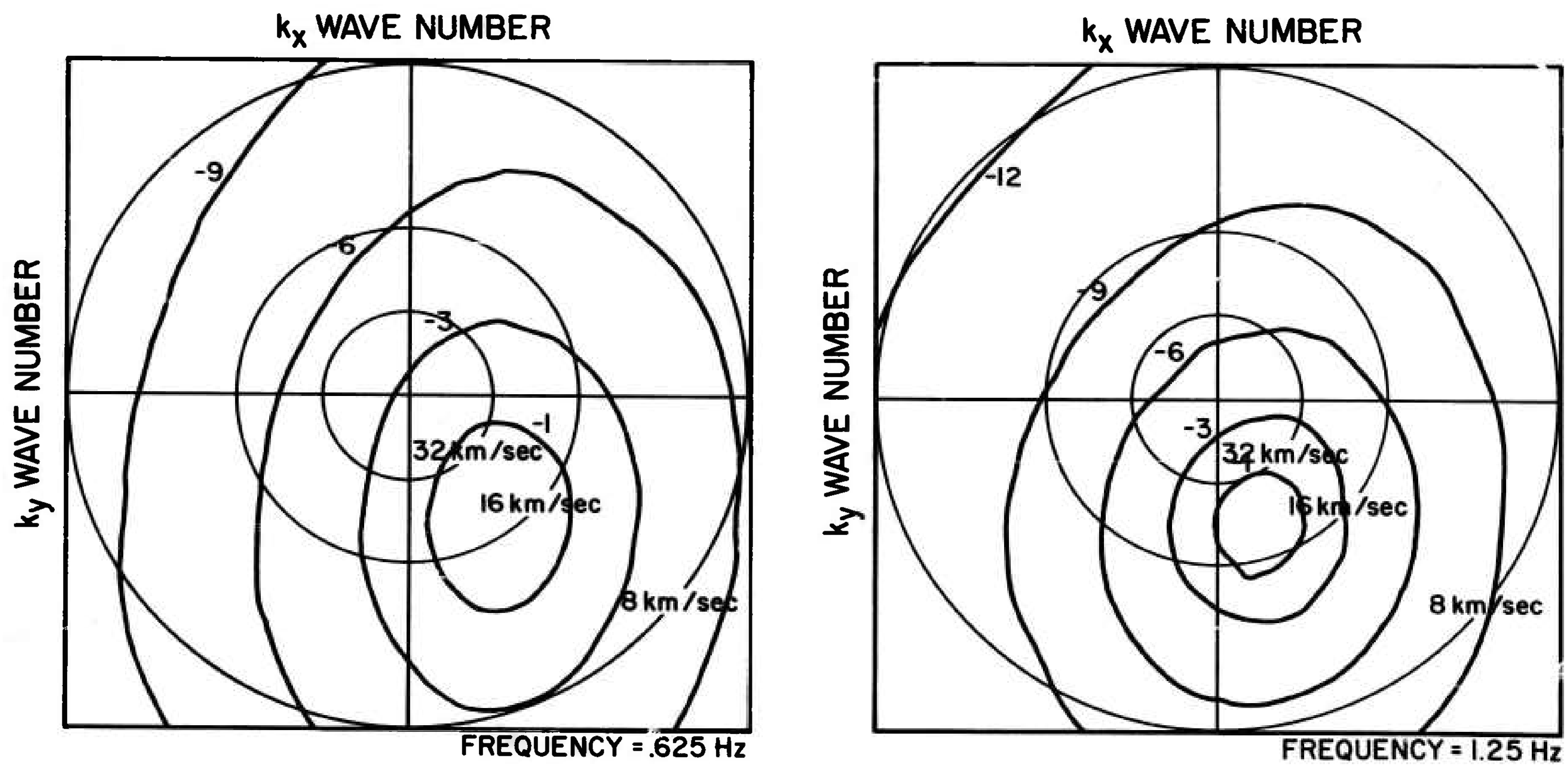

Figure 7. High-resolution frequency-wavenumber spectra of a WMSO signal

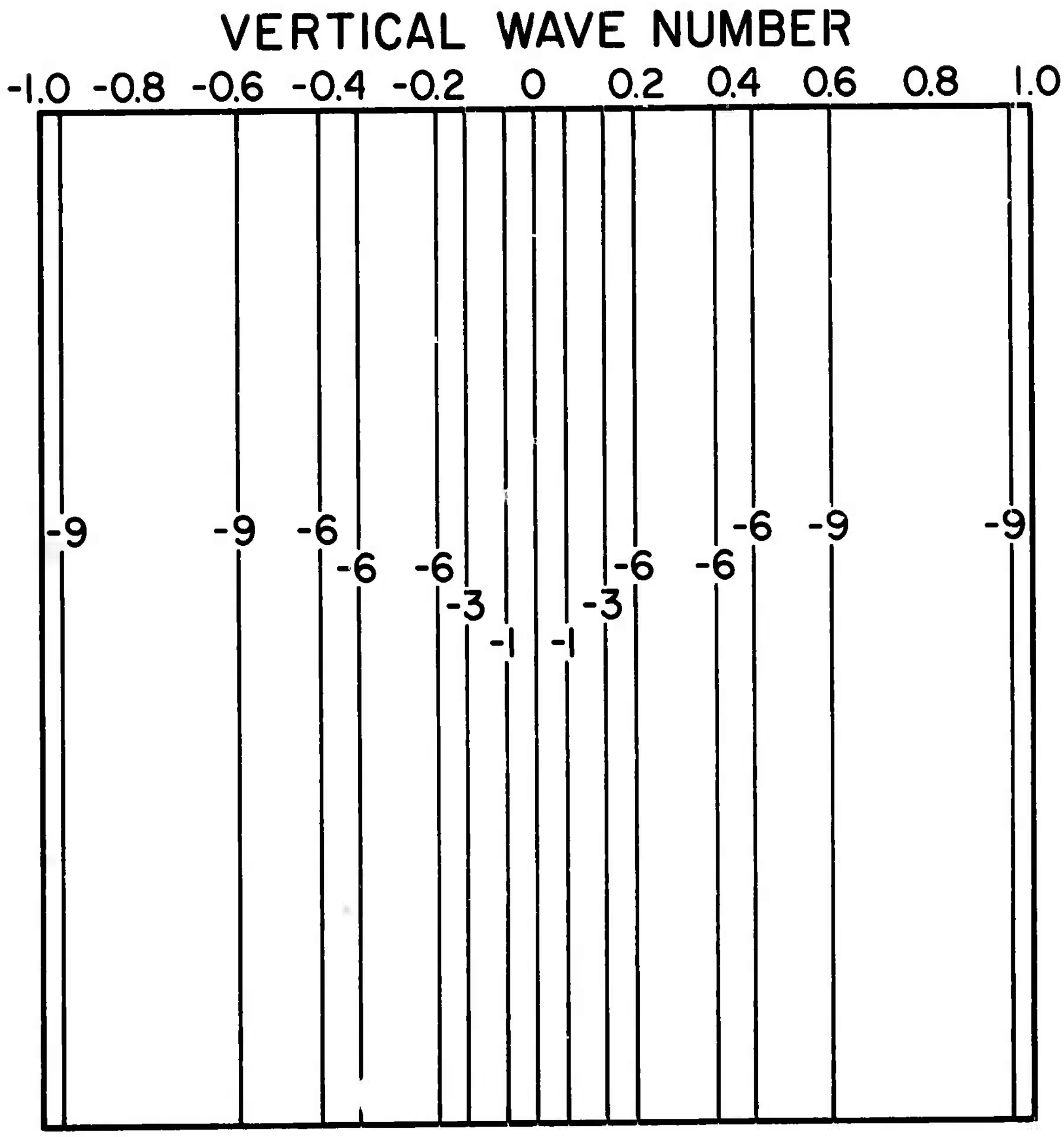

Figure 8. APOK array response

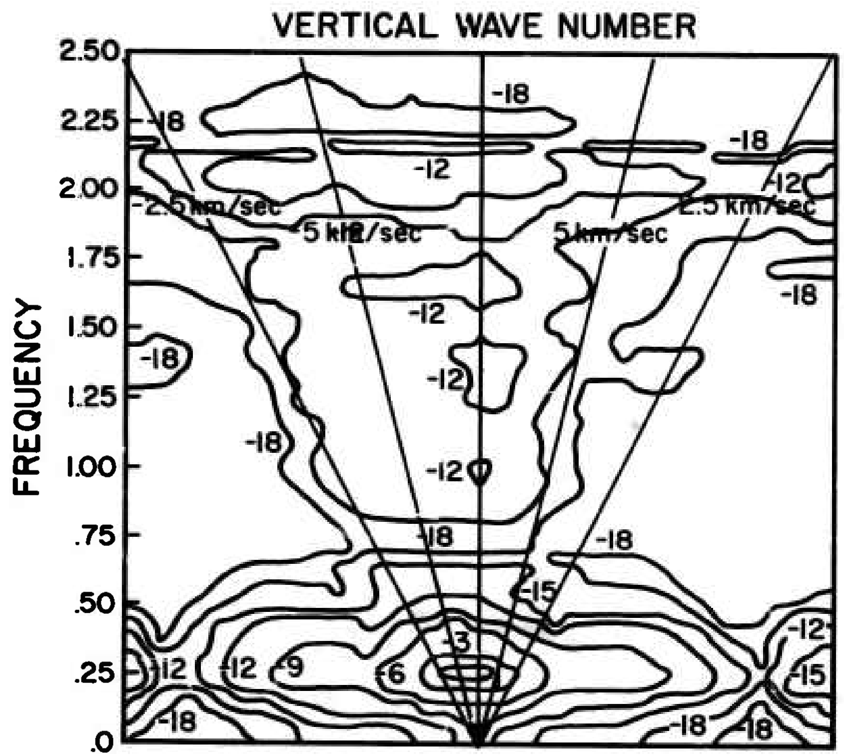

Figure 9a. Ordinary frequency-wavenumber spectrum of APOK noise

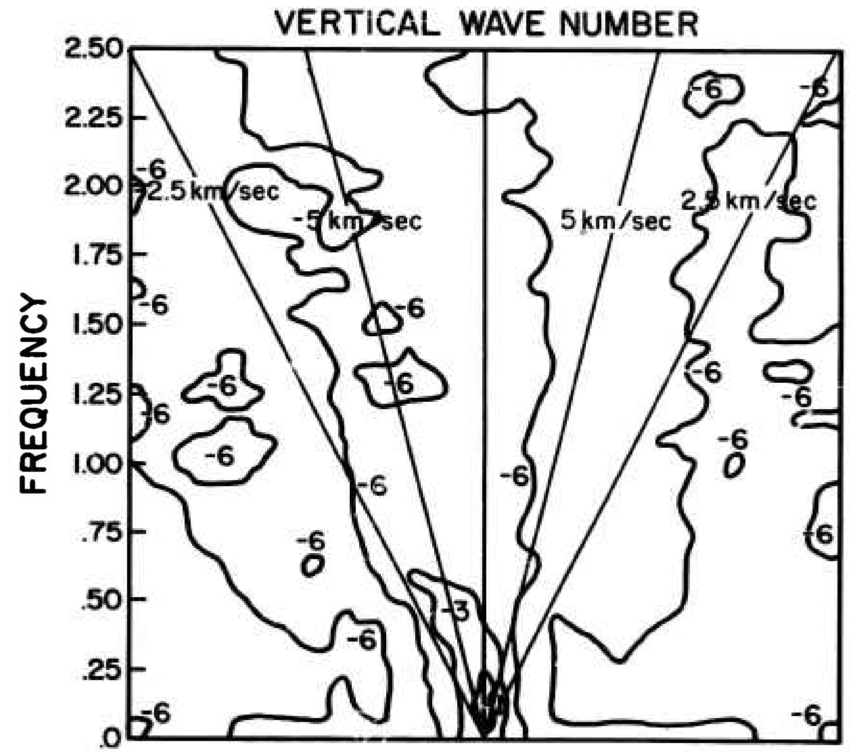

Figure 9b. High-resolution frequency-wavenumber spectrum of APOK noise

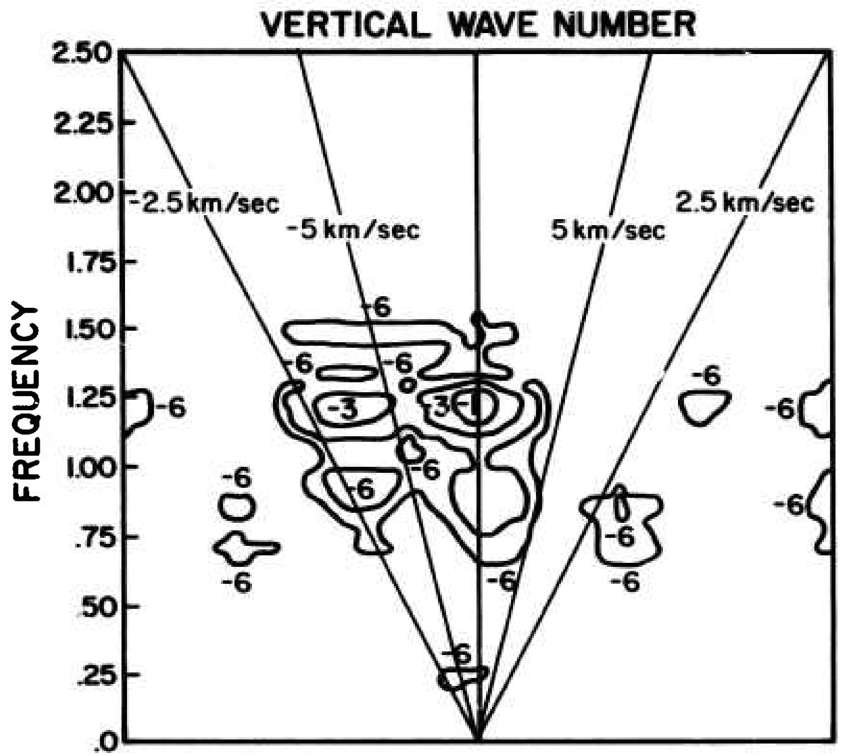

Figure 10a. Ordinary frequency-wavenumber spectrum of an APOK signal

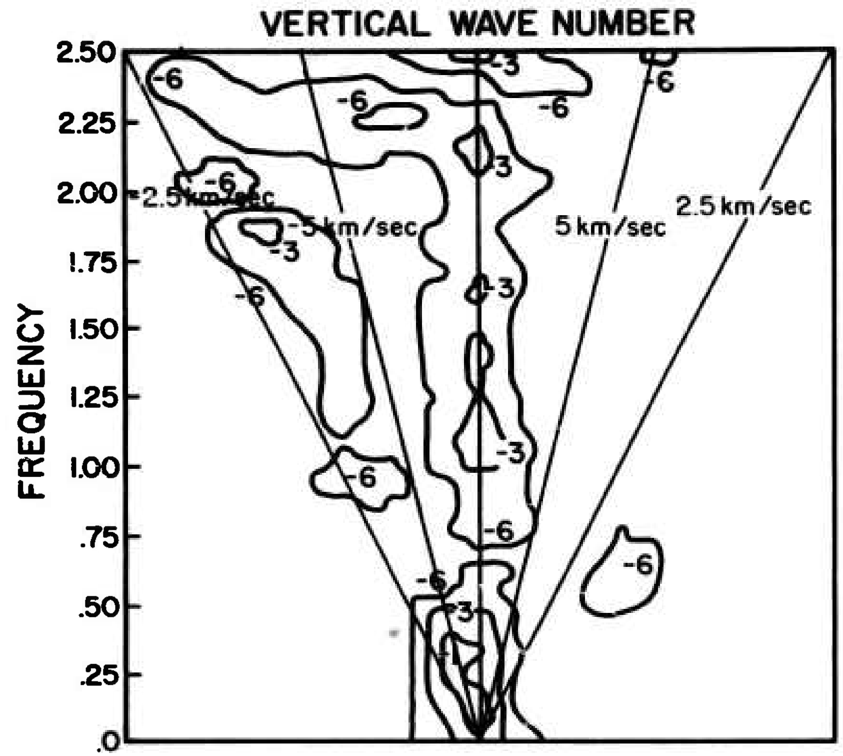

Figure 10b. High-resolution frequency-wavenumber spectrum of an APOK signal

Table 1

EVENT DATA

	WMSO Event	APOK Event
Latitude	41.4° S.	52.0° N.
Longitude	88.6° W.	175.0° W.
Origin Time	14:51:55.0	19:05:04.1
Depth (km)	33	67
Magnitude	4.9	5.2
Region	W. Chile Rise	Aleutians

Data taken from USCGS Preliminary Determination of Epicenters

Table 2

SENSOR COORDINATES

WMSO SURFACE ARRAY

Sensor	X(km.)	Y(km.)
V_1	0.00	0.00
V_2	0.67	0.74
V_3	0.94	-0.17
V_4	0.27	-0.70
V_5	-0.99	0.12
V_6	-0.34	0.71
V_7	0.08	1.41
V_8	1.35	1.33
V_9	1.68	0.34
V_{10}	1.19	-1.02
V_{11}	-0.54	-0.78
V_{12}	-1.51	-0.59
V_{13}	-1.29	0.95

APOK VERTICAL ARRAY

Sensor	Depth (km.)
DW_5	0.02
DW_4	1.68
DW_3	1.99
DW_2	2.29
DW_1	2.91

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